Parameter Estimation based Optimal control for a Bubble Cap Distillation Column


School of Electrical and electronics, SASTRA University, Thanjavur, India.

*School of Chemical & Biotechnology, SASTRA University, Thanjavur, India.

*Corres.author: aruchemxl@scbt.sastra.edu

Abstract: Distillation column is one of the key elements in the process of chemical industries, which is having nonlinear, multivariable and non-stationary characteristics. The conventional controller like PID provides ineffective control action for nonlinear system. This paper deals the design of the model based predictive controller to control composition of the Bubble cap distillation column consists of a combination of methanol and water. The estimation of parameters is used to build the exact model of the process. The MATLAB platform is used in the implementation of MPC and conventional PID.

Keywords: Distillation column, System identification, Model predictive control, PID.

I. Introduction

Distillation is one of the best separations technic in a chemical and petroleum industries and tight control action is important from the plant economics\(^1\). The purpose of the distillation system is to separate a liquid mixture into two or more components. Distillation Column modelling and controlling both very difficult because it’s dealing with multivariable, nonlinear and non-stationary process. The composition control plays the vital role in distillation columns. Generally innovative process control tools increase the flexibility and performance of the chemical plants. The conventional controller (PID) employed to control the distillation column does not guarantee tight control action because it’s highly nonlinear\(^2\). To solve critical control issues and to achieve better performance in industrial application, PID controllers are used\(^3\text{-}^5\) but they face difficulties in controlling non-linear process and cannot predict immediate change in an input. To overcome these difficulties MPC controller is used and it is mainly used for industries side. Actually the distillation column mathematical model needs to be implemented the predictive controller so that here the real time data will be taken from the distillation column and the model will be developed from with the help of system identification technic. Fuzzy logic based model and control approach applied\(^6\) and neural network employed to both the model and identification for distillation column and it has been used to fuzzy-neural based inferential control\(^8\) but all the scenarios did not provided any scope of optimization technic. S. Joe Qin has discussed about industries use of MPC\(^9\). Some of the tutorial papers\(^10\) by James B. Rawlings help to gain insights in MPC. Some review articles
consider MPC on academic perspective. Some paper deal with (SMPC) simplified model predictive control algorithm\textsuperscript{11}. The custom of step response model Dynamic matrix control (DMC) increases the computational load\textsuperscript{12}. Generalized predictive controller (GPC) is the most popular controller and it’s generally used it can be accept the state space representation models and reduce the computational time\textsuperscript{13}.

This paper ordered as follows the system identification and experimental structure are clarified in section (II) and Section (III) deals with controller synthesis, finally the section (IV) deal with results and discussion.

II. System Identification and Experimental Structure

Design a linear model based controller, first the appropriate linear models have to be identified and validated\textsuperscript{14}. Linear model can be obtained by two ways one is system identification and another is linearization of a nonlinear model. Both approaches have different advantages and disadvantages. System identification techniques used through experiment study is possible but the nonlinear model of the process having different open loop and closed loop studies as possible. This paper describes linear model based model predictive control design using system identification techniques. Linear black box models can be obtained by ARX, ARMA, ARMAX, ARARMAX, ARARX etc., These black box models can be developed by correlating sequence relationship between input and output data, here the input is reboiler temperature (manipulated variable) and the output is overhead product composition (controlled variable). Fig (1) shows that the structure of a distillation column. The real time data are taken from by using a reflux rate of the column as kept constant and to give the sudden step changes of the reboiler temperature. Fig (1) shows that structure of a distillation column. After obtaining the data model has been developed by using a least square algorithm (LS). The main application of least squares is model fitting. Least square technic is mainly used for estimating the system parameter and minimization of error so that here the system parameter estimation and error minimization developed by least square method it is one of the system identification technic.

![Fig. 1. Structure of Distillation column](image)

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The general equation of the least square algorithm is given by

\begin{align*}
    y(k) + \alpha_1 y(k-1) + \alpha_2 y(k-2) &= \beta_1 u(k) + \beta_2 u(k-1) \\
    y(0) + \alpha_1 y(-1) + \alpha_2 y(-2) &= \beta_1 u(-1) + \beta_2 u(0) \\
    y(1) + \alpha_1 y(0) + \alpha_2 y(-1) &= \beta_1 u(1) + \beta_2 u(0) \\
    y(2) + \alpha_1 y(1) + \alpha_2 y(0) &= \beta_1 u(2) + \beta_2 u(1) \\
    \vdots \\
    y(n) + \alpha_1 y(n-1) + \alpha_2 y(n-2) &= \beta_1 u(n) + \beta_2 u(n-1)
\end{align*}

(1)

System parameters are given by

\[ \theta = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \end{bmatrix} \]

(2)

Output values are

\[ y = \begin{bmatrix} y(1) \\ \vdots \\ y(n) \end{bmatrix} \]

(3)

Recursive vectors are given by

\[ w = \begin{bmatrix} \psi^T(0) \\ \psi^T(1) \\ \vdots \\ \psi^T(N-1) \end{bmatrix} = \begin{bmatrix} -Y(0) & -Y(-1) & U(1) & U(0) \\ -Y(1) & -Y(0) & U(2) & U(1) \\ \vdots & \vdots & \vdots & \vdots \\ -Y(N-1) & -Y(N-2) & U(N-1) & U(N-2) \end{bmatrix} \]

(4)

\[ Y = \varphi \theta \]

(5)

E is the N-dimensional error vectors

\[ e = y_d - y \]

\[ e^T = (e(1), e(2), \ldots, e(n)) \]

Performance measure of J is given by

\[ J = e^T e = \sum_{k=1}^{N} e^2(k) \]

\[ J = (y - \varphi \theta)^T (y - \varphi \theta) \]

(6)

To optimize the performance measure, parameters are estimated

\[ \frac{\partial J}{\partial \theta} = -2 \varphi^T (y - \varphi \theta) \]
III. Generalized Predictive Controller

The MPC provides various algorithms and best algorithm is Generalized Predictive Algorithm (GPC). MPC is one of the advanced control strategies, which can forecast the future response of the plant and optimize the control input with the help of a model of the plant. The prediction model will be augmented by the model of state space matrices Fig (2) shows the structure of MPC.

![Fig.2. Structure of MPC](image)

The augmented matrix given as

\[
\begin{bmatrix}
\Delta x_m(k+1) \\
y(k+1)
\end{bmatrix} = \begin{bmatrix}
\Delta x_m(k) \\
y(k)
\end{bmatrix} + \begin{bmatrix}
\hat{\beta} \\
\hat{\alpha}
\end{bmatrix} \Delta u(k)
\]

(8)

\[
y(k) = \begin{bmatrix}
o_{m1} & 1
\end{bmatrix} \begin{bmatrix}
x_m(k) \\
y(k)
\end{bmatrix}
\]

(9)

Where

\[
o_{m1} = \begin{bmatrix}
o_{m1} & 1
\end{bmatrix}_{m1}
\]

\(\alpha_m, \beta_m\) and \(\gamma_m\) are represented by the plant parameters. \(\Delta u(k_1) + \cdots + \Delta u(k_i + N_c - 1)\) are represented by the future control signals. Here the \(N_c\) represents the control horizon and \(N_p\) represents the prediction horizon. The future state variables are estimated as

\[
x(k_i + 1|k_i) = \alpha x(k_i) + \beta \Delta u(k_i)
\]

\[
x(k_i + 2|k_i) = \alpha^2 x(k_i) + \alpha \beta \Delta u(k_i) + \beta \Delta u(k_i + 1)
\]
\[ x(k_i + N_p|k_i) = \alpha N_p x(k_i) + \alpha N_p^{-1} \beta \Delta u(k_i) + \ldots + \alpha N_p^{-N_p} \beta \Delta u(k_i + N_p - 1) \] (10)

The future output is,
\[ y(k_i + 1|k_i) = \gamma \alpha x(k_i) + \gamma \beta \Delta u(k_i) \]
\[ y(k_i + 2|k_i) = \gamma \alpha^2 x(k_i) + \gamma \alpha \beta \Delta u(k_i) + \gamma \beta \Delta u(k_i + 1) \]
\[ \vdots \]
\[ y(k_i + N_p|k_i) = \gamma \alpha N_p x(k_i) + \gamma \alpha N_p^{-1} \beta \Delta u(k_i) + \ldots + \gamma \alpha N_p^{-N_p} \beta \Delta u(k_i + N_p - 1) \] (11)

From the eqn (4), output generalized use
\[ Y = Fx(k_i) + \Theta \Delta u \] (12)

Where
\[ F = \begin{bmatrix} \gamma \alpha \\ \gamma \alpha^2 \\ \vdots \\ \gamma \alpha N_p \end{bmatrix}_{(N_p \times 1)} \]

And
\[ \Theta = \begin{bmatrix} \gamma \beta & 0 & 0 & \ldots & 0 \\ \gamma \alpha \beta & \gamma \beta & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma \alpha N_p \beta & \gamma \alpha N_p^{-2} \beta & \gamma \alpha N_p^{-3} \beta & \ldots & \gamma \alpha N_p^{-N_p} \beta \end{bmatrix}_{(N_p \times N_p)} \] (13)

Eqn (6) and Eqn (7) further used to minimize the cost function. The main objective is predicted output is near as possible to the set point. \( \Delta U \) is mainly used to change the control signal and it should find the error among predicted output and the set point is minimized
\[ R^2 = \begin{bmatrix} 1 & 1 & \ldots & 1 \end{bmatrix} r(kk) \] (14)

Here we assume the set point is constant and the cost function J is defined by
\[ J = (R_x - Y)^T (R_x - Y) + U^T R U \] (15)

\[ R = \eta_{w|Np|Nc} \] Where the \( \eta_w \) is tuning parameter.

Substituting the output (Y) equation and we get
\[ J = (R_x - Fx(k_i))^T (R_x - Fx(k_i)) - 2 \Delta U^T \Psi^T (R_x - Fx(k_i)) + \Delta U^T (\Phi^T \Phi + R) \Delta U \] (16)

Here our objective cost function is minimized and we get J is respect to \( \Delta U \)
\[ \Delta U = (\Phi^T \Phi + R)^{-1} \Phi^T (R_x - Fx(k_i)) \] (17)
IV. Results and Discussion

The real time data are taken from the experimental distillation column fig (3) and fig (4) shows that response of input and output of the process and fig (5) for the model validation curve. Here the GPC values are tuned by Sridhar and cooper tuning method\textsuperscript{15}. The PID is adjusted by the Ziegler Nichols (Z-N) method. Both the PID controller and GPC controller for the Bubble cap distillation column validated using MATLAB environment and the result is obtained. The GPC controller tuning strategies are shown in Table (1) and the PID control tuning parameters are shown in Table (2) and then the performance indices in tabulated in Table (3). The response graph shown in fig (6) and the different step changes response shown in the fig (7), fig (8) and the response to the positive disturbance response and the negative disturbance response plotted in the fig (9), fig (10) from the responses we prove that GPC gives fast response and quick setting time of the PID.

![Fig.3.Process reaction curve of reboiler temperature](image1)

![Fig.4.Process reaction curve for composition](image2)
Fig. 5. Model validation curve

Fig. 6. Comparison of the response of PID and GPC

Fig. 7. Positive set point changes response comparison of PID and GPC
Fig. 8. Negative set changes response comparison of PID and GPC

Fig. 9. Comparison of positive disturbance in PID and GPC

Fig. 10. Comparison of negative disturbance in PID and GPC
V. Conclusion

In this work GPC is designed and control a composition of a Bubble cap distillation column and its response compared with a PID. The comparison has been done between GPC and PID, it shows that MPC provided better performance than PID by observing ISE, ITAE and IAE.

Table 1: Tuning The Parameters Of MPC

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Np</td>
<td>10</td>
</tr>
<tr>
<td>Nc</td>
<td>7</td>
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<tr>
<td>T</td>
<td>1</td>
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Table 2: Tuning The Parameters Of PID

<table>
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<tr>
<td>P</td>
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<tr>
<td>I</td>
<td>0.8</td>
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<tr>
<td>D</td>
<td>0.01</td>
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Table 3: Performance Measure Characteristics

<table>
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<tr>
<th>Controller</th>
<th>ISE</th>
<th>IAE</th>
<th>ITAE</th>
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<tr>
<td>PID</td>
<td>178.2970</td>
<td>387.700</td>
<td>4.9245</td>
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<td>GPC</td>
<td>78.7254</td>
<td>175.2852</td>
<td>1.8220</td>
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References


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