

# Optimization of Control Parameters Using Improved Relay Tuning and Taguchi Method

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**Abstract:** The first order plus time delay (FOPTD) model was considered and the improved relay tuning method considering higher order of harmonics (N) was applied for tuning to calculate the corrected ultimate gain (Ku). This corrected value was used to estimate PID parameters by Ziegler-Nichols (ZN) method. Then these PID parameters were considered for Taguchi orthogonal array (OA) and their three different values were considered. The Taguchi Method was applied to estimate the optimum values of PID parameters. The closed loop response was obtained using these optimum values of PID parameters and compared with that of conventional method and Padmasree method. It was found that the Integral Square Error (ISE) for conventional, Padmasree method and proposed method are 27.43, 27 and 7.9 respectively and hence the proposed method gives better performance.

**Keywords:** Relay Tuning, Taguchi Method, FOPTD, Simulink.

## Introduction:

Control plays a key role in the operation of chemical plants with respect to economical performance, safety and operability. In a typical chemical plant there are hundreds of PID feedback loops. They are often poorly tuned because the choice of PID controller parameters requires professional knowledge by the user. One of the most common approaches to tune a controller automatically is to connect a relay as a feedback controller to the process during tuning. **Astrom and Hagglund**<sup>1</sup> have suggested the use of an ideal (on-off) relay to generate a sustained oscillation of the controlled variable and to get the ultimate gain ( $k_u$ ) and the ultimate frequency ( $\omega_u$ ) directly from the relay experiment. The relay feedback method has become very popular because, it is time efficient as

compared to the conventional method. The amplitude ( $a$ ) and the period of oscillation ( $p_u$ ) are noted from the sustained oscillation of the system output. The ultimate gain ( $K_u$ ) and ultimate frequency ( $\omega_u$ ), are calculated from the principal harmonics approximation as given by equation;

$$K_u = \frac{4h}{\pi a} \quad (1)$$

$$\omega_u = \frac{2\pi}{P_u} \quad (2)$$

**Luyben**<sup>2</sup> has suggested the use of relay testing for identifying a transfer function model. Using  $k_u$  and  $\omega_u$  in the phase angle and amplitude criteria for an unstable FOPTD model, the following two equations relating three model parameters are obtained

$$\frac{K_u K_p}{(1 + \tau^2 \omega_u^2)^{0.5}} = 1 \quad (3)$$

$$D\omega_u - \tan^{-1}(\tau\omega_u) = 0 \quad (4)$$

Since only  $k_u$  and  $\omega_u$  are available, additional information such as the steady state gain, or the time delay should be a known priori in order to fit a typical transfer function model such as unstable FOPTD. The above Equations assume that, the higher order harmonics are neglected.

**Thyagarajan and Yu<sup>3</sup>** have proposed a method of identifying a FOPTD unstable model based on the shape of the response of the process using a symmetric relay. In this method, the output response is aligned with the input response by shifting to the left. Then, the time to peak amplitude, the peak amplitude and the period of oscillation are noted. The time delay is considered as the time to the peak value. From the derived analytical expression of the process output response of an unstable FOPTD system for a symmetric relay input, the time constant and gain are calculated as;

$$\tau = \frac{\frac{P_u}{2}}{\ln\left(\frac{1}{2e^{D/\tau} - 1}\right)} \quad (5)$$

$$K_p = \frac{a}{h(e^{D/\tau} - 1)} \quad (6)$$

It is to be noted that, for higher order systems, the recorded time to peak value from the response ( $D$ ) will not match with that of the actual time delay of the process. **Li, Eskinat, and Luyben<sup>4</sup>** have reported that the model identified by the symmetry relay auto tune method gives error as high as **27 to -18%** in the value of  $k_u$  for stable FOPTD systems. Recently, **Srinivasan and Chidambaram<sup>5</sup>** proposed a method of considering higher order harmonics, to explain the reported error of 27 to -18% in  $k_u$  calculations for stable systems. **Sathe Vivek, M. Chidambaram<sup>6</sup>** proposed an improved method by incorporating the higher order harmonics to explain the error in the  $K_u$  calculation. The relay equation is given as;

$$y(t) = a[1 + (1/9) + (1/25) + (1/49) + (1/81) + \dots] \quad (7)$$

### **Taguchi's Robust Tuning Method:**

The Taguchi's robust parameter design is used to determine the levels of factors and to minimize the sensitivity to noise. That is, a parameter setting should be determined with the intention that the

product response has minimum variation while its mean is close to the desired target. Taguchi's method is based on statistical and sensitivity analysis for determining the optimal setting of parameters to achieve robust performance<sup>7</sup>. In setting up a framework for robust design, the classifications of the quantities at play in the design task are given. Design Variables (DV) are those quantities to be decided by the designer with the purpose of meeting performance specifications under given conditions. Design-Environment Parameters (DEP) is those quantities over which the designer has no control and that define the conditions of the environment under which the designed object will operate. Performance Functions (PF) are quantities used to represent the performance of the design in terms of design variables and design-environment parameters. The responses at each setting of parameters are treated as a measure that would be indicative of not only the mean of some quality characteristic, but also the variance of the same characteristic. The mean and the variance are combined into a single performance measure known as the Signal-to-Noise (S/N) ratio<sup>7,8</sup>. Taguchi classifies robust parameter design problems into different categories depending on the goal of the problem and for each category as follows:

**Smaller the better:** The target value of  $y$ , that is, quality variable is zero. In this situation, S/N Ratio (SNR) is defined as follows

$$SNR = -10 \log \left( \frac{1}{n} \sum_{i=1}^n y_i^2 \right) \quad (8)$$

**Larger the better:** The target value of  $y$ , that is, quality variable is infinite and S/N ratio is defined as follows:

$$SNR = -10 \log \left( \frac{1}{n} \sum_{i=1}^n \frac{1}{y_i^2} \right) \quad (9)$$

**Nominal the best:** The certain target value ( $s$ ) is given for  $y$  value. In this situation S/N ratio is defined as follows:

$$SNR = -10 \log \left( \frac{1}{n} \sum_{i=1}^n \frac{y_i^2}{s^2} \right) \quad (10)$$

In this paper, an improved relay tuning is applied to calculate corrected ultimate gain and then the Taguchi Method is applied to estimate the optimum values of PID parameters.

### Materials and Method:

A process having the following FOPTD transfer function model was considered;

$$G_p = \frac{e^{-0.2}}{s-1} \quad (11)$$

Then the Simulink diagram for relay tuning was prepared as shown in fig 1. The relay experiments were carried out for relay height as 0.2 and N as 5. The amplitude (a), period of oscillations (Pu) was noted and corrected ultimate gain was calculated.

The PID controller tuning parameters were calculated using Ziegler-Nichols<sup>9</sup> optimum controller parameter method as shown in Table 1. Then these PID parameters i.e. Kc,  $\tau_I$  &  $\tau_D$  were chosen as three

parameters and their three different levels were considered as shown in Table 2. An orthogonal array of L9 was chosen for the analysis. The experiments were carried out for each level and the Integral Square Error (ISE) were calculated for each experiment. Then the SNR were calculated for each experiment based on smaller the better characteristics. The sum of SNR was calculated for each parameter and at each level as shown in Table 3. The optimum level of each parameter i.e. the level which contains maximum value of sum of the SNR is the optimum level, were noted. Then using these optimum values of PID parameters, the closed loop response was studied and it was compared with that of the conventional method (using h=1 & N=1) and Padmasree method<sup>10</sup>.

**Table 1: Ziegler-Nichols optimum controller parameter**

Controller	Kc	$\tau_I$	$\tau_D$
P	0.5Ku	-	-
P+I	0.45Ku	Pu/1.2	-
P+I+D	0.6Ku	Pu/2	Pu/8

**Table 2: OA for PID parameters.**

Kc	$\tau_I$	$\tau_D$	ISE	Fr. ISE	SNR
4	0.3	0	8.325	0.0832	10.97
4	0.4	0.1	27.261	0.2726	5.64
4	0.5	0.2	27.315	0.2731	5.63
4.2	0.3	0.1	27.532	0.2753	5.6
4.2	0.4	0.2	27.947	0.2794	5.53
4.2	0.5	0	8.208	0.082	10.86
4.4	0.3	0.2	28.066	0.28	5.52
4.4	0.4	0	8.622	0.0862	10.64
4.4	0.5	0.1	27.133	0.2713	5.66

**Table 3: Sum of SNR for different factors and levels for PID Tuning**

Parameter	Sum of SNR at each level			Total
	L1	L2	L3	
Kc	<b>22.24</b>	21.99	21.82	66.05
$\tau_I$	22.09	21.81	<b>22.15</b>	66.05
$\tau_D$	<b>32.47</b>	16.9	16.68	66.05

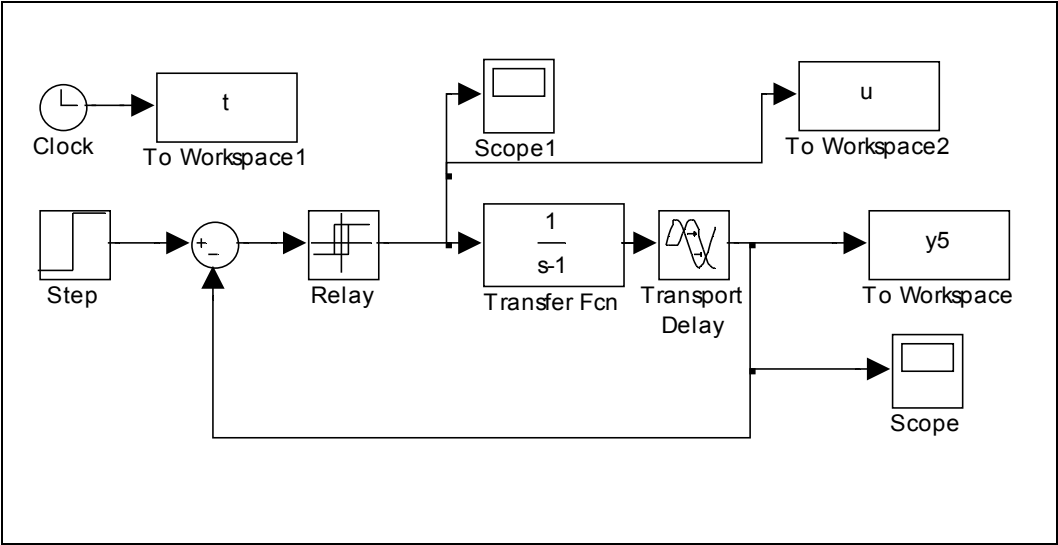


Fig 1: Simulink Diagram for Relay Control.

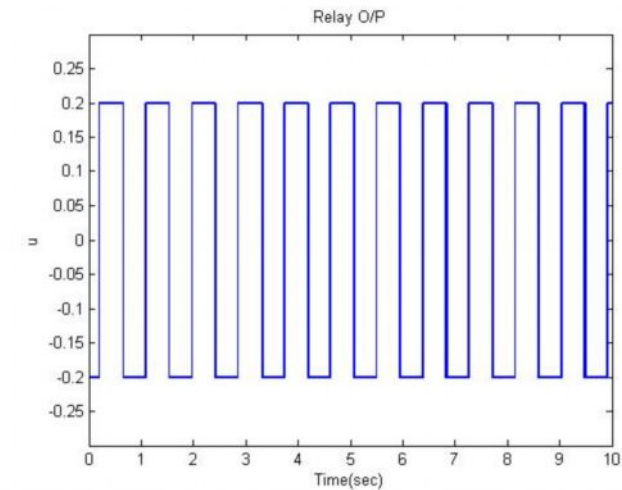


Fig 2: Relay Response

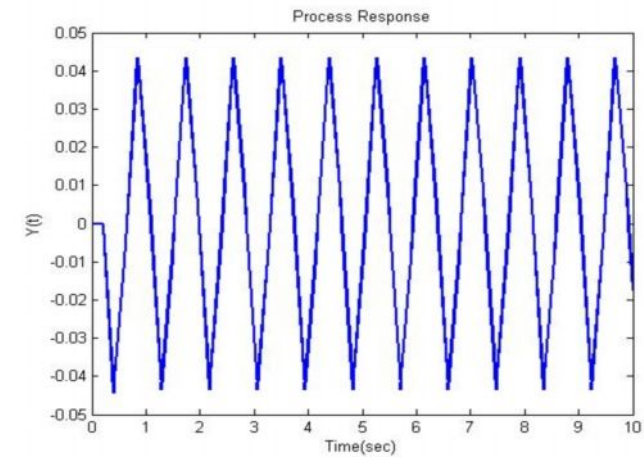


Fig 3: Process Response

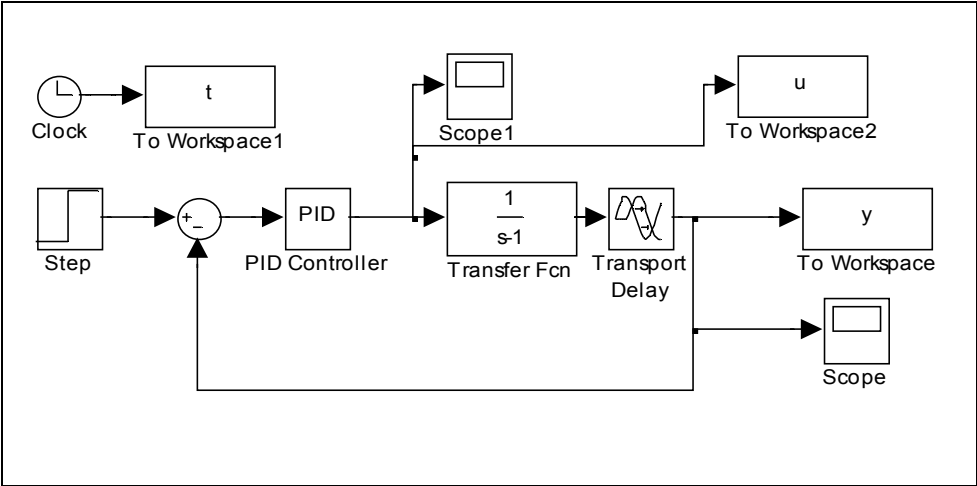


Fig 4: Simulink Diagram for PID Control.

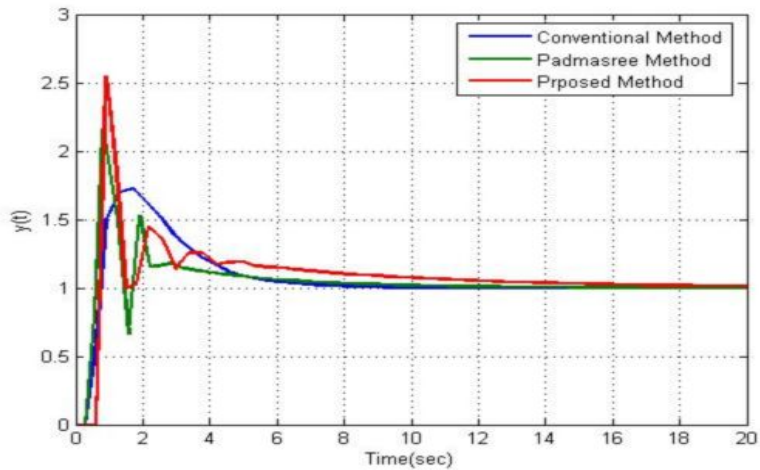


Fig 5: Closed Loop Response with optimum values of PID parameters.

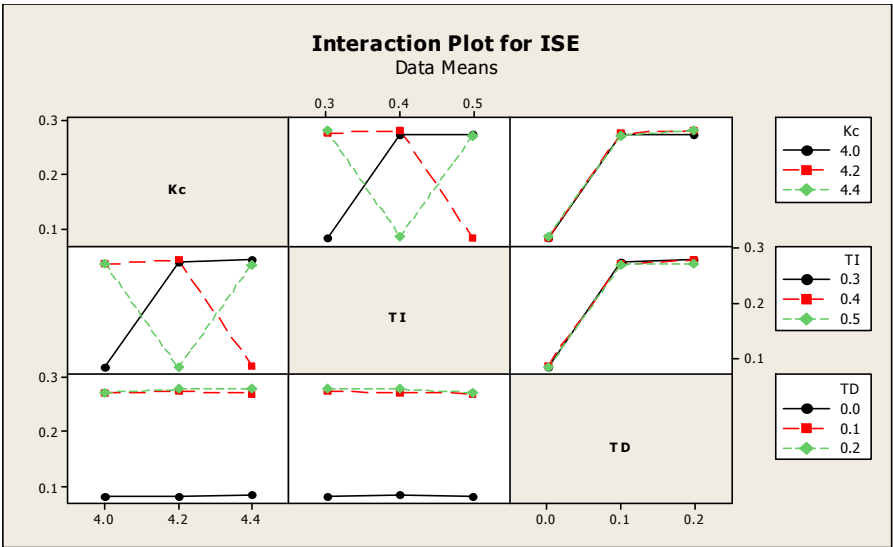


Fig 6: Interaction plot for ISE

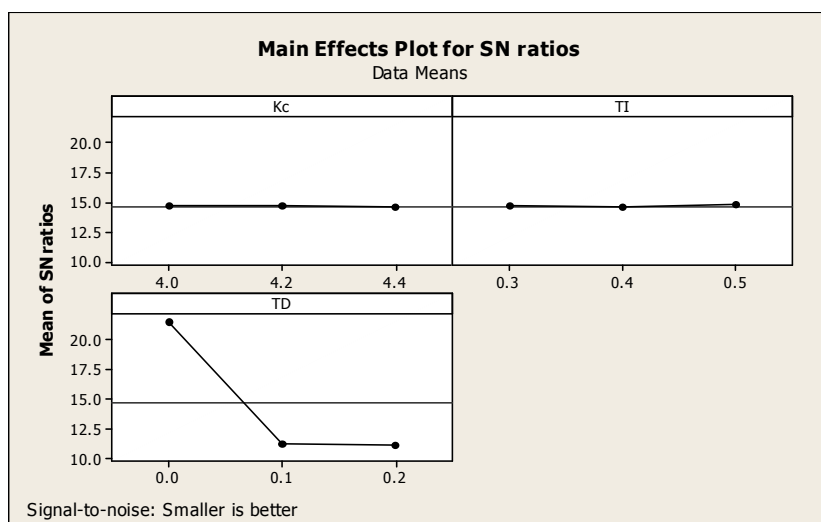


Fig 7: Main Effects Plot

### Result & Discussion:

For improved relay tuning, two parameters i.e. relay height ( $h$ ) as 0.2 and number of harmonics ( $N$ ) as 5 was considered for experimentation. The experiments were conducted and the corrected ultimate gain ( $K_u$ ) was calculated. The actual value of  $K_u$  by amplitude criteria is 7.22. And the calculated  $K_u$  by conventional and improved relay method are 5.86 & 7 respectively. Thus the error in calculation of  $K_u$  obtained by conventional method is 18% and that by improved relay method has been reduced up to 3%. The Simulink diagram for experiment, relay response and process response are shown in figure 1-3 respectively. The PID parameters were calculated using Ziegler-Nichols<sup>9</sup> optimum controller parameter method as shown in Table 1. The estimated PID parameters found to be  **$K_c=4.2$ ;  $\tau_I = 0.4$  and  $\tau_D=0.1$ .**

For Taguchi method, three parameters i.e.  $K_c$ ,  $\tau_I$  and  $\tau_D$  were considered and their three different values were considered for experimentation. An orthogonal array of L9 was chosen for the analysis. The experiments were carried out for each level and the ISE were calculated for each experiment. Then the SNR were calculated for each experiment based on smaller the better characteristics. The sum of SNR was calculated for each parameter and at each level as shown in table 3. The Simulink diagram for experiment is shown in figure 4. The optimum values for each parameter were noted. That is, the level which contains maximum value of sum of the SNR is the optimum level. The optimum values found to be  $K_c=4$ ;  $\tau_I = 0.5$ ;  $\tau_D = 0$  where that of conventional method gives

$K_c=2.63$ ;  $\tau_I = 0.73$ ;  $\tau_D=0.18$  and Padmasree method gives  $K_c=4.89$ ;  $\tau_I = 1.1$ ;  $\tau_D=0.1$ .

Using these optimum values, the closed loop response was obtained and it was compared with that of conventional method and Padmasree method<sup>10</sup> as shown in fig. 5.

### Conclusion:

In this study, an improved Relay-tuning PID control scheme for FOPTD process control along with Taguchi's method is presented. By using this scheme, the parameters are optimally and robustly adjusted with respect to the system dynamics. This technique is found to be more effective than conventional tuning methods. This method can be easily extended to multi input and multi-output systems from basic single-input and single-output systems. The simple structure, robustness and ease of computation of the proposed method make it very attractive for real time implementation for control of given process. The proposed method gives the best performance. It was found that the Integral Square Error (ISE) for conventional, Padmasree method and proposed method are 27.43, 27 and 7.9 respectively and hence the proposed method gives better performance.

### Appendix A

Equation used for PID settings (Padmasree et al., 2004):

$$\begin{aligned} \text{Let } \varepsilon &= D/\tau, \\ k_{cp} &= 1.2824\varepsilon^{-0.8325} \quad \text{for } 0.01 \leq \varepsilon \leq 1.2, \\ \tau_I/\tau &= 5.573\varepsilon - 0.0063 \quad \text{for } 0.01 \leq \varepsilon \leq 0.5, \\ \tau_I/\tau &= 0.483 \exp(-3.3739\varepsilon) \quad \text{for } 0.5 \leq \varepsilon \leq 1.2, \\ \tau_D/\tau &= 0.507\varepsilon + 0.0028 \quad \text{for } 0.01 \leq \varepsilon \leq 1.2. \end{aligned}$$

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