



## **Application of Graph Theoretic Approach in Selection of a Car**

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**Abstract :** The present day automotive industry is a buyers' market. The process of buying a car has transformed into a problem of buying a car. The proposed study develops a framework for selection of car using Graph theory matrix approach. The selection of attributes and sub attributes were done based on literature review and experts opinion. The attributes digraph was developed and the same was represented in matrix form. The permanent function was used to find the Index score. The option with most elevated index score was observed to be the best option. This methodology will help any individual without much technical knowledge in selecting a car.

**Keywords :** Graph theoretic approach – Digraph - Permanent function - Index score.

### **Introduction:**

Buying a car is one's bigger decisions, as there are many things to consider. The alluring looks and performance of cars made the selection of car even more complicated. The problem of selection of car is addressed as a Multi criteria decision making problem in the literature. In <sup>1</sup> adapted fuzzy TOPSIS method to select an automobile. In <sup>2</sup> proposed AHP method to analyze the consumer preferences in selection of a luxury car. In <sup>3</sup> proposed a framework in selecting an automobile using an extension of AHP. In <sup>4</sup> structured a fuzzy analytic network process for selection of automobile. In <sup>5</sup> adapted MACBETH and Multi MOORA method in selection of an automobile. In <sup>6</sup> focused on ranking of cars using integrated fuzzy ANP with PROMETHEE and GRA. The evaluation of automobiles was done using a model based on integrated AHP and TOPSIS by <sup>7</sup>.

The proposed study is centered on investigating the different variables that impact the determination in selection of a car and build up a decision making method for selecting the best alternative using Graph theoretic approach. The capacity to show the criteria connections and the capacity to produce various leveled models empowers the Graph theoretic way to deal with tackle complex problems <sup>8,9</sup>.

### **Graph Theoretic Approach:**

Graph theory is proved to be beneficial for solving real life problems in the field of science and technology <sup>10, 11</sup> and it maintains the hierarchical structure of the system and also utilizes the inter relations among the attributes <sup>12</sup>.

The step by step procedure of Graph theoretic approach along with the application is explained as follows:

**Step 1:** Identify the alternatives. The alternatives selected in this study are Hyundai Elite i20 ERA 1.4 CRDi, Maruti suzuki swift DLX diesel, Ford Figo Ambiente 1.5 TDCi, Maruti Suzuki Baleno 1.3 Sigma, Hyundai Grand i10, CRDi Sportz celebration edition, Toyota Etios Liva GD, Chevrolet Beat Diesel PS, Volkswagen Polo GT TDI, Maruti Celerio LDI, Toyota Etios Cross GD, Fiat Punto Evo 1.3 Emotion and Renault PULSE RxL ABS and are designated as C1, C2, C3, C4, C5, C6, C7, C8, C9, C10, C11 and C12 respectively.

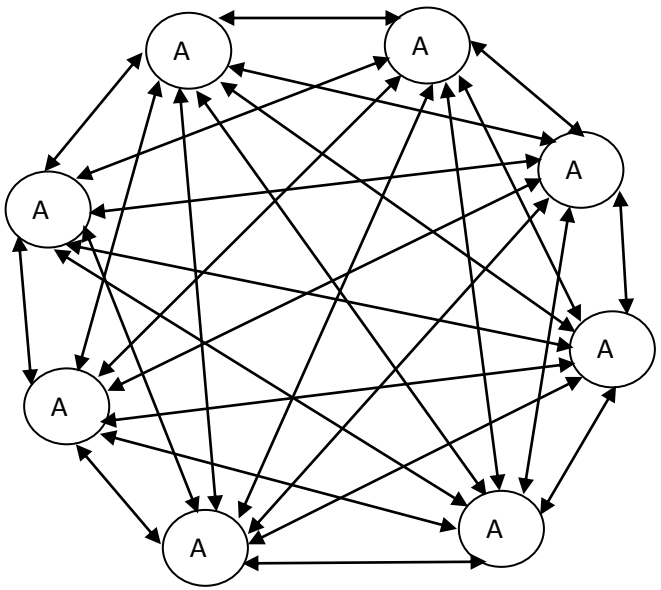
**Step 2:** The attributes and sub-attributes are identified and selected which influence the selection of alternatives. Table 1 shows the details of attributes and sub attributes.

**Table. (1): List of attributes and sub attributes.**

S. No	Attributes	Sub attributes
1	Engine (A1)	Engine Displacement in CC (S1)
		Power in HP (S2)
		Speed for the rated power in rpm (S3)
		No Of Cylinders (S4)
		Kerb Weight in kgs. (S5)
2	Performance (A2)	Overall mileage in Km/l (S6)
		Top Speed in Km/h (S7)
		Minimum Turning Radius in m (S8)
		Wheel Size in Inch (S9)
3	Suspension (A3)	Suspension Front (S10)
		Suspension Rear (S11)
		Brakes Front (S12)
		Steering Type (S13)
4	Interior (A4)	Seat Upholstery (S14)
		No of Seating Rows (S15)
		Boot Space in litres (S16)
		Fuel Capacity in litres (S17)
5	Exterior (A5)	Length in mm (S18)
		Width in mm (S19)
		Height in mm (S20)
		Wheelbase in mm (S21)
		Ground Clearance in mm (S22)
6	Comfort (A6)	Air- conditioning (S23)
		Adjustable Steering (S24)
		Power Windows (S25)
		Adjustable Driver Seat (S26)
		Seat Belt Warning (S27)
7	Safety (A7)	Airbags (S28)
		Anti-lock Braking System (S29)
		Collapsible Steering Column (S30)
		Fog Lamps -Front / Rear (S31)

		Rear Wash Wiper (S32)
8	Overall (A8)	Price in Lakhs (S33)
		Customer rating (S34)
		Standard Warranty in Years (S35)
		Standard Warranty in kilometers (S36)

**Step 3:** Thedigraphs for attributes and sub attributes are plotted. On the chance that a node has a significance on another node then, a directed edge is drawn between the nodes. The digraph for attributes i.e., Engine, Performance, Suspension, Interior, Exterior, Comfort, Safety, Overall is shown in Figure 1.



**Fig. (1): Attributes digraph**

**Step 4:** Convert the attributes digraph into the matrix form. The diagonal elements of the matrix represent the individual importance of the attribute. The attributes matrix [G], for Fig 1 is given as,

$$[G] = \begin{bmatrix} D1 & d12 & d13 & d14 & d15 & d16 & d17 & d18 \\ d21 & D2 & d23 & d24 & d25 & d26 & d27 & d28 \\ d31 & d32 & D3 & d34 & d35 & d36 & d37 & d38 \\ d41 & d42 & d43 & D4 & d45 & d46 & d47 & d48 \\ d51 & d52 & d53 & d54 & D5 & d56 & d57 & d58 \\ d61 & d62 & d63 & d64 & d65 & D6 & d67 & d68 \\ d71 & d72 & d73 & d74 & d75 & d76 & D7 & d78 \\ d81 & d82 & d83 & d84 & d85 & d86 & d87 & D8 \end{bmatrix}$$

**Step 5:** Ascribe weights to inheritance and relative importance in the matrix. Table 2 may be used to assign the values of relative importance<sup>10, 13</sup>.

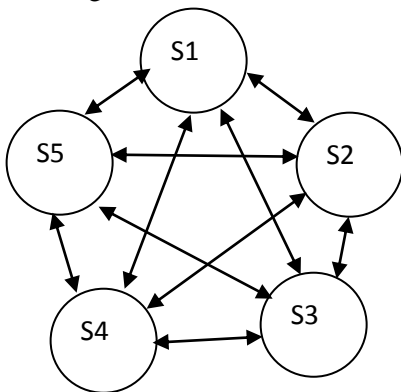
**Table. (2): Relative importance values**

S. No.	Class description	Relative importance	
		$a_{ij}$	$a_{ji} = 1 - a_{ij}$
1	Two attributes are equally important	0.5	0.5
2	One attribute (i) is slightly more important over the other (j)	0.6	0.4
3	One attribute (i) is strongly important over the other (j)	0.7	0.3
4	One attribute (i) is very strongly important over the other (j)	0.8	0.2
5	One attribute (i) is extremely important over the other (j)	0.9	0.1
6	One attribute (i) is exceptionally more important over the other (j)	1.0	0.0

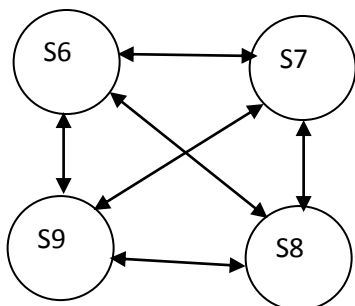
The values of diagonal elements i.e the inheritance may be obtained as follows:

**Step 5.1:** Identify the sub attributes for the selected attributes. Table 1 shows the details of attributes and sub attributes.

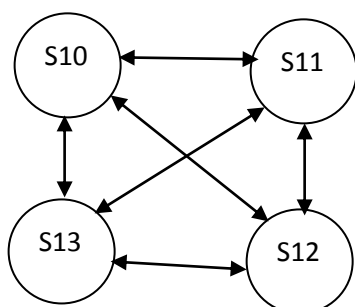
**Step 5.2:** Plot the sub attributes digraphs. Figure 2, Figure 3, Figure 4, Figure 5, Figure 6, Figure 7, Figure 8 and Figure 9 shows the sub attributes digraphs for the considered attributes.



**Figure 2. Digraph for sub attribute Engine**



**Figure 3. Digraph for sub attribute Performance**



**Figure 4. Digraph for sub attribute Suspension**

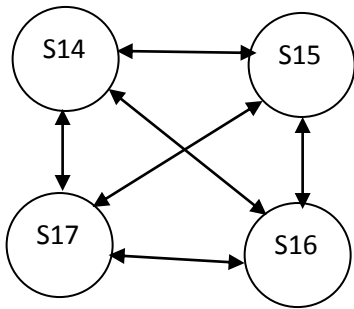


Figure 5. Digraph for sub attribute Interior

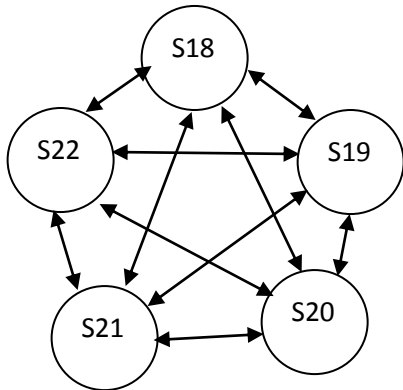


Figure 6. Digraph for sub attribute Exterior

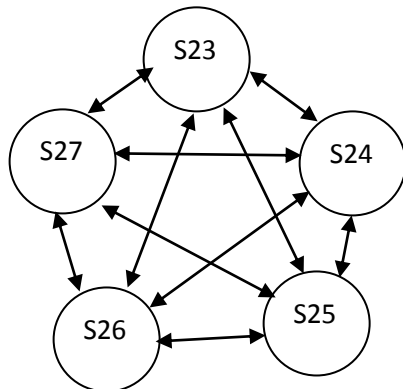


Figure 7. Digraph for sub attribute Comfort

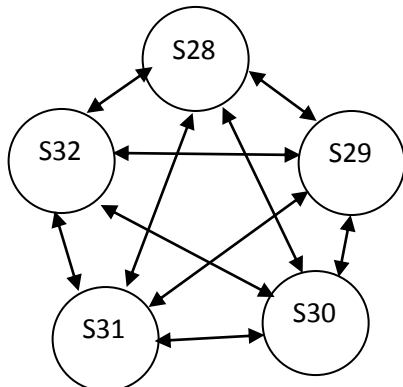
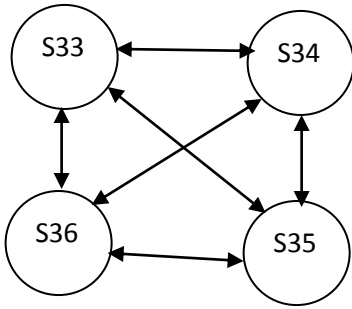


Figure 8. Digraph for sub attribute Safety



**Figure 9. Digraph for sub attribute Overall**

**Step 5.3:** Convert the sub attributes digraphs into respective matrices.

The matrix for sub attributes for attribute Engine is given as,

$$[A1] = \begin{bmatrix} S1 & a12 & a13 & a14 & a15 \\ a21 & S2 & a23 & a24 & a25 \\ a31 & a32 & S3 & a34 & a35 \\ a41 & a42 & a43 & S4 & a45 \\ a51 & a52 & S5 & a54 & S5 \end{bmatrix}$$

The matrix for sub attributes for attribute Performance is given as,

$$[A2] = \begin{bmatrix} S6 & a12 & a13 & a14 \\ a21 & S7 & a23 & a24 \\ a31 & a32 & S8 & a34 \\ a41 & a42 & a43 & S9 \end{bmatrix}$$

The matrix for sub attributes for attribute Suspension is given as,

$$[A3] = \begin{bmatrix} S10 & a12 & a13 & a14 \\ a21 & S11 & a23 & a24 \\ a31 & a32 & S12 & a34 \\ a41 & a42 & a43 & S13 \end{bmatrix}$$

The matrix for sub attributes for attribute Interior is given as,

$$[A4] = \begin{bmatrix} S14 & a12 & a13 & a14 \\ a21 & S15 & a23 & a24 \\ a31 & a32 & S16 & a34 \\ a41 & a42 & a43 & S17 \end{bmatrix}$$

The matrix for sub attributes for attribute Exterior is given as,

$$[A5] = \begin{bmatrix} S18 & a12 & a13 & a14 & a15 \\ a21 & S19 & a23 & a24 & a25 \\ a31 & a32 & S20 & a34 & a35 \\ a41 & a42 & a43 & S21 & a45 \\ a51 & a52 & a53 & a54 & S22 \end{bmatrix}$$

The matrix for sub attributes for attribute Comfort is given as,

$$[A6] = \begin{bmatrix} S23 & a12 & a13 & a14 & a15 \\ a21 & S24 & a23 & a24 & a25 \\ a31 & a32 & S25 & a34 & a35 \\ a41 & a42 & a43 & S26 & a45 \\ a51 & a52 & a53 & a54 & S27 \end{bmatrix}$$

The matrix for sub attributes for attribute Safety is given as,

$$[A7] = \begin{bmatrix} S28 & a12 & a13 & a14 & a15 \\ a21 & S29 & a23 & a24 & a25 \\ a31 & a32 & S30 & a34 & a35 \\ a41 & a42 & a43 & S31 & a45 \\ a51 & a52 & a53 & a54 & S32 \end{bmatrix}$$

The matrix for sub attributes for attribute Overall is given as,

$$[A8] = \begin{bmatrix} S33 & a12 & a13 & a14 \\ a21 & S34 & a23 & a24 \\ a31 & a32 & S35 & a34 \\ a41 & a42 & a43 & S36 \end{bmatrix}$$

The normalized values of inheritance are shown in Table 3 and the linguistic terms in sub attributes are ascribed with suitable values.

**Table. (3): Details of inheritance for sub attributes**

Attributes and Sub attributes / Alternatives	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
<b>Engine</b>												
Engine Displacement (CC)	1197	1197	1498	1248	1120	1364	936	1498	793	1364	1248	1461
Power	81.83	83.11	99	74	70	67.04	56.3	103.5	47	67.06	91.7	63.1
Rated power at Speed (rpm)	6000	6000	3750	4000	4000	3800	4000	4400	3500	3800	4000	4000
No Of Cylinders	4	4	4	4	3	4	3	4	2	4	4	4
Kerb Weight (kgs)	1066	1415	1100	960	1025	995	1027	1148	880	1015	1198	1060

<b>Performance</b>												
Overall (Km/l)	18.5	20.4	25.83	21.4	21.2	23.59	25.44	19.91	27.62	23.59	21.2	23
Top Speed (Km/h)	170	165	170	160	157	180	165	183	130	160	165	160
Minimum Turning Radius (m)	5.2	4.8	4.9	4.9	4.8	4.8	4.85	4.97	4.7	4.8	5	4.65
Wheel Size (Inch)	14	14	14	15	14	14	14	15	13	15	15	14
<b>Suspension</b>												
Suspension Front	M1	M3	M2	M3	M3	M3	M3	M3	M3	M3	M3	M3
Suspension Rear	A2	A1	A3	A1	A2	A1	A2	A3	A2	A1	A1	A1
Brakes Front	D1	D2	D2	D1	D1	D2	D1	D1	D2	D2	D2	D2
Steering Type	E1	E1	E1	E1	E1	E1	E2	E1	E3	E1	E4	E1
<b>Interior</b>												
Seat Upholstery	F1	F1	F1	F1	F1	F1	F1	F1	F1	F1	F1	F1
No of Seating Rows	2	2	2	2	2	2	2	2	2	2	2	2
Boot Space (litres)	295	204	257	339	256	251	170	280	235	251	280	251
Fuel Capacity (litres)	45	42	40	37	43	45	35	45	35	45	45	41
<b>Exterior</b>												
Length	3985	3850	3886	3995	3765	3775	3640	3971	3600	3895	3989	3805
Width	1734	1695	1695	1745	1660	1695	1595	1682	1600	1735	1687	1665
Height	1505	1530	1525	1500	1520	1510	1520	1469	1560	1555	1525	1525
Wheelbase (mm)	2570	2430	2491	2520	2425	2460	2375	2469	2425	2460	2510	2450
Ground Clearance (mm)	170	170	174	170	165	170	175	165	165	174	195	154
<b>Comfort</b>												
AC	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Adjustable Steering	N	T1	T1	T1	T1	T1	N	T2	N	T1	T1	T1
Power Windows	F	B	F	F	N	Y	N	B	N	B	B	B
Adjustable Driver Seat	N	Y	M	N	N	Y	N	M	N	Y	M	M
Seat Belt Warning	Y	Y	Y	Y	Y	Y	Y	N	N	Y	Y	Y
<b>Safety</b>												
Airbags	2	N	1	2	0	2	9	2	2	2	2	1
Anti-lock Braking System	N	N	N	Y	N	Y	N	Y	Y	Y	Y	Y
Collapsible Steering Coloum	N	Y	Y	N	N	Y	Y	Y	Y	Y	N	Y



Fog Lamps	N	F	R	N	F	N	N	B	N	F	B	N
Rear Wash Wiper	N	N	N	N	N	N	N	Y	N	Y	Y	Y
<b>Overall</b>												
Price	5.69	4.54	6.03	6.57	6.79	6.78	5.09	9.33	5.17	7.89	8.04	6.23
customer ratings	4	4.2	4	4.4	3.9	4.1	3.8	4.4	3.7	4	3.9	3.8
Standard Warranty (Years)	2	2	2	1	1	3	3	2	2	3	3	2
Standard Warranty (kilometers)	Unlimited	40000	100000	40000	Unlimited	100000	100000	Unlimited	40000	100000	100000	50000

**Step 5.4:** The inheritance and relative importance are substituted in Eq. 1 and the permanent function for all matrices of sub attributes are assessed.

$$\begin{aligned}
 Per(A) = & \prod_{i=1}^M D_i + \sum_{i=1}^{M-1} \sum_{j=i+1}^M \dots \sum_{M=i+1}^M (d_{ij} d_{ji}) D_k D_l D_m D_n D_o \dots D_t D_m \\
 & + \sum_{i=1}^{M-2} \sum_{j=i+1}^{M-1} \sum_{k=j+1}^M \dots \sum_{M=i+1}^M (d_{ij} d_{jk} d_{ki} + d_{ik} d_{kj} d_{ji}) D_l D_m D_n D_o \dots D_t D_m \\
 & + [\sum_{i=1}^{M-3} \sum_{j=i+1}^M \sum_{k=i+1}^{M-1} \sum_{l=i+2}^M \dots \sum_{M=i+1}^M (d_{ij} d_{ji}) (d_{kl} d_{lk}) D_m D_n D_o \dots D_t D_m + \\
 & \sum_{i=1}^{M-3} \sum_{j=i+1}^{M-1} \sum_{k=i+1}^M \sum_{l=i+1}^M \dots \sum_{M=i+1}^M (d_{ij} d_{jk} d_{kl} d_{li} + d_{il} d_{lk} d_{kj} d_{ji}) D_m D_n D_o \dots D_t D_m ] \\
 & + [\sum_{i=1}^{M-2} \sum_{j=i+1}^{M-1} \sum_{k=j+1}^M \sum_{l=1}^{M-1} \sum_{m=l+1}^M \dots \sum_{M=i+1}^M (d_{ij} d_{jk} d_{ki} + d_{ik} d_{kj} d_{ji}) (d_{lm} d_{ml}) D_n D_o \dots D_t D_m \\
 & + \sum_{i=1}^{M-4} \sum_{j=i+1}^{M-1} \sum_{k=i+1}^M \sum_{l=i+1}^M \sum_{m=j+1}^M \dots \sum_{M=i+1}^M (d_{ij} d_{jk} d_{kl} d_{lm} d_{mi} + d_{im} d_{ml} d_{lk} d_{kj} d_{ji}) D_n D_o \dots D_t D_m ] \\
 & + [\sum_{i=1}^{M-3} \sum_{j=i+1}^M \sum_{k=i+1}^M \sum_{l=j+1}^{M-1} \sum_{m=n+1}^M \dots \sum_{M=i+1}^M (d_{ij} d_{jk} d_{kl} d_{li} + d_{il} d_{lk} d_{kj} d_{ji}) (d_{mn} d_{nm}) D_o \dots D_t D_m \\
 & + \sum_{i=1}^{M-5} \sum_{j=i+1}^{M-1} \sum_{k=j+1}^M \sum_{l=1}^{M-2} \sum_{m=l+1}^M \sum_{n=m+1}^M \dots \sum_{M=i+1}^M (d_{ij} d_{jk} d_{ki} + d_{ik} d_{kj} d_{ji}) (d_{lm} d_{mn} d_{nl} + d_{ln} d_{nm} d_{ml}) D_o \dots D_t D_m \\
 & + \sum_{i=1}^{M-5} \sum_{j=i+1}^M \sum_{k=i+1}^M \sum_{l=i+2}^M \sum_{m=k+1}^M \sum_{n=k+2}^M \dots \sum_{M=i+1}^M (d_{ij} d_{ji}) (d_{kl} d_{lk}) (d_{mn} d_{nm}) D_o \dots D_t D_m \\
 & + \sum_{i=1}^{M-5} \sum_{j=i+1}^{M-1} \sum_{k=i+1}^M \sum_{l=i+1}^M \sum_{m=i+1}^M \sum_{n=j+1}^M \dots \sum_{M=i+1}^M (d_{ij} d_{jk} d_{kl} d_{lm} d_{mn} d_{ni} + d_{in} d_{nm} d_{ml} d_{lk} d_{kj} d_{ji}) D_o \dots D_t D_m ] \\
 & + \dots
 \end{aligned}
 \tag{1}$$

A computer program is used to evaluate the values of permanent function. The permanent function values for sub attributes are shown in Table 4. It is to be noted that these values are taken as inheritance for the attributes.

**Table. (4): Values of permanent function**

	Engine (A1)	Performance (A2)	Suspension (A3)	Interior (A4)	Exterior (A5)	Comfort (A6)	Safety (A7)	Overall (A8)
C1	0.834	0.962	0.665	1.286	1.282	0.056	0.008	0.705
C2	1.085	0.944	0.534	0.912	1.184	0.974	0.036	0.315
C3	1.183	1.157	0.921	1.039	1.241	0.663	0.045	0.604
C4	0.782	1.028	0.455	1.209	1.265	0.095	0.016	0.263
C5	0.584	0.933	0.556	1.104	1.108	0.054	0.016	0.500

C6	0.767	1.116	0.534	1.133	1.168	0.663	0.037	0.891
C7	0.460	1.109	0.693	0.685	1.086	0.031	0.017	0.704
C8	1.448	1.097	0.455	1.233	1.150	0.140	1.506	1.046
C9	0.228	0.896	1.006	0.864	1.065	0.009	0.037	0.309
C10	0.781	1.082	0.534	1.133	1.272	0.974	1.237	0.970
C11	1.115	1.062	0.675	1.233	1.407	0.974	0.238	0.964
C12	0.846	0.976	0.534	1.043	1.065	0.974	0.138	0.408

**Step 6:** The inheritance and inter relationships of sub attributes are used to repeat step 3 to step 5 to evaluate the permanent function for the attributes considered.

**Step 7:** The values of permanent function for the attributes are tabulated in Table 5 and sort to rank them. This permanent function values is also said as the Index score.

**Table. (5): Index scores and rank of alternatives**

S. No	Name	Designation	Index score	Rank
1	Hyundai Elite i20 ERA 1.4 CRDi	C1	0.23787	8
2	Maruti suzuki swift DLX diesel	C2	0.38775	7
3	Ford Figo Ambiente 1.5 TDCi	C3	0.78979	4
4	Maruti Suzuki Baleno 1.3 Sigma	C4	0.12383	11
5	Hyundai Grand i10 CRDi Sportz celebration edition	C5	0.13225	10
6	Toyota Etios Liva GD	C6	0.64147	5
7	Chevrolet Beat Diesel PS	C7	0.13561	9
8	Volkswagen Polo GT TDI	C8	2.03485	3
9	Maruti Celerio LDI	C9	0.0912	12
10	Toyota Etios Cross GD	C10	4.71123	1
11	Fiat Punto Evo 1.3 Emotion	C11	2.43916	2
12	Renault PULSE RxL ABS	C12	0.55017	6

**Step 8:** Selection of best option. The option with the most elevated Index score is observed to be the best option.

## Conclusion

This study presented an application of Graph theory matrix approach in selection of a car. The elements that influence the selection of car are taken as attributes and sub attributes. The digraphs were developed for attributes and sub attributes. The permanent function concept was adapted such that there is no loss of information among the inheritance and relative importance. The Index score evaluated using the permanent function was used to select the best alternative.

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## Nomenclature

- A1 Torsion beam
- A2 Coupled torsion beam
- A3 Semi-independent twist beam
- D1 Disc

D2	Ventilated disc
E1	Rack & pinion
E2	Electric power steering
E3	Electronic power steering
E4	Hydraulic power steering
F1	Fabric
M1	Mc pherson strut with coil spring
M2	Independent strut with coil spring
M3	Mc pherson strut

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